



Semester One Examination, 2016

Question/Answer Booklet

**MATHEMATICS
SPECIALIST
UNIT 1**

**Section Two:
Calculator-assumed**

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: ten minutes
Working time for section: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in the WACE examinations

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	13	13	100	102	65
Total				154	100

Instructions to candidates

- The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
- Write your answers in this Question/Answer Booklet.
- You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
- Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
 - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
 - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.
- Show all your working clearly.** Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- It is recommended that you **do not use pencil**, except in diagrams.
- The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

Section Two: Calculator-assumed

65% (102 Marks)

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 100 minutes.

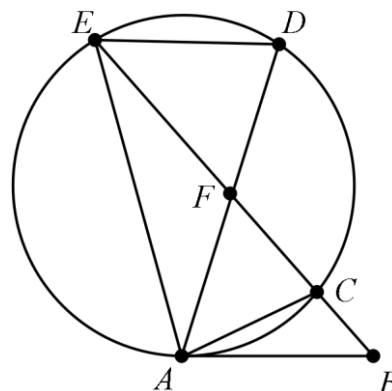
Question 8

(5 marks)

In the diagram below, AB is a tangent to the circle and $AB = BF = FD$.

Triangle AEB is similar to triangle CAB

Triangle FDE is similar to triangle FCA



If $BC = 4$ and $CE = 32$, determine the lengths of

(a) AB . (2 marks)

(b) CF . (1 mark)

(c) AF . (2 marks)

Question 9

(7 marks)

Two forces act on body. The first has magnitude 250 N and acts in direction 240° and the second has magnitude 410 N and acts in direction 170° .

- (a) By writing each force in component form, determine the resultant of the two forces.
(4 marks)

- (b) The work done, in joules, by a force in moving a body is the scalar product of the resultant force, in newtons, and the displacement, in metres. Determine the total work done by the two forces, if the body moves 45 metres in direction 215° .
Hint: Write the displacement in component form.
(3 marks)

Question 10

(8 marks)

(a) Use a counterexample to demonstrate that each of following statements are false.

(i) $ab = ac, \{a, b, c \in \mathbb{Z}\} \Rightarrow b = c.$ (2 marks)

(ii) If $f(n) = n^2 + n + 17, n \in \mathbb{Z}$, then $f(n)$ is always prime. (2 marks)

(b) The statement 'if a natural number is a multiple of 4 and 5 then the natural number is a multiple of 20' is true.

(i) Write the contrapositive of the statement and explain whether or not the contrapositive is also true. (2 marks)

(ii) Write the converse of the statement and explain whether or not the converse is also true. (2 marks)

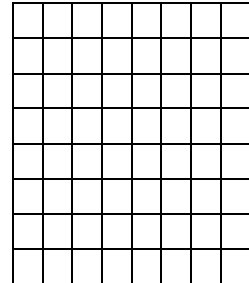
Question 11

(9 marks)

(a) Consider the 8×8 chessboard below.

(i) How many rectangles of all sizes does the board contain?

(2 marks)



(ii) How many of these rectangles are squares?

(3 marks)

(b) Consider the digits 1, 2, 3, 4, 5, 6, 7 and 9. If no digit may be repeated and the numbers include 2 odd digits and 2 even digits?

(i) How many 4-digit numbers can be formed?

(2 marks)

(ii) How many such numbers are even?

(2 marks)

Question 12

(7 marks)

Vector \mathbf{a} has magnitude 6 units and acts on a bearing of 310° . Vector \mathbf{b} has magnitude 12 units and acts on a bearing of 070° .

(a) Determine the magnitude and direction of $3\mathbf{a} + 2\mathbf{b}$. (4 marks)

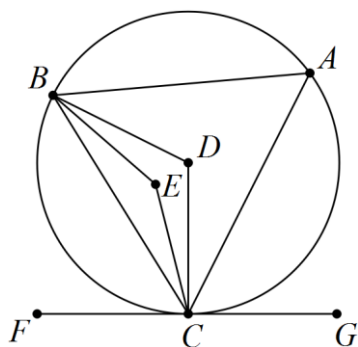
(b) Determine the value of the constant k if the direction of $5\mathbf{a} + k\mathbf{b}$ is due north. (3 marks)

Question 13

(9 marks)

- (a) AB is a diameter of a circle centre O . C is a point on the circumference. D is a point on AC such that OD bisects $\angle AOC$. Prove that OD is parallel to BC . (4 marks)

- (b) In the diagram below, FCG is a tangent to the circle ABC . BD bisects $\angle ABC$, CD bisects $\angle ACB$, BE bisects $\angle DBC$ and CE bisects $\angle DCB$. If $AB = AC$ and $\angle ACG = 48^\circ$, determine the ratio, in simplest form, of $\angle BAC : \angle BDC : \angle BEC$. (5 marks)



Question 14

(9 marks)

Points O , P , Q and R have position vectors $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$, $\begin{pmatrix} 15 \\ y \end{pmatrix}$, $\begin{pmatrix} x \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 3 \\ -5 \end{pmatrix}$.

(a) Determine the value of y if $|\mathbf{OP}| = 17$.

(2 marks)

(b) Determine the value of x if $\overrightarrow{OQ} \perp \overrightarrow{QR}$.

(3 marks)

(c) Determine the values of x and y if R lies on the line between P and Q such that $PR : RQ = 1 : 2$.

(4 marks)

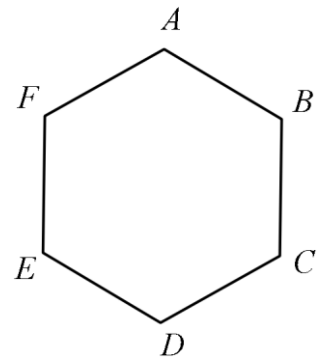
Question 15

(9 marks)

- (a) $ABCDEF$ is a regular hexagon in which \overrightarrow{BC} represents \mathbf{b} and \overrightarrow{FC} represents $2\mathbf{a}$.

Express the vectors \overrightarrow{CD} , \overrightarrow{EA} and \overrightarrow{BE} in terms of \mathbf{a} and \mathbf{b} .

(3 marks)



- (b) Given \mathbf{c} and \mathbf{d} are vectors such that $|\mathbf{c}| = 4$, $|\mathbf{d}| = 6$ and the angle between their directions is 40° , determine

(i) $|4\mathbf{c} + 3\mathbf{d}|$.

(3 marks)

(ii) the angle between \mathbf{d} and $\mathbf{d} - 3\mathbf{c}$.

(3 marks)

Question 16

(10 marks)

- (a) If a room contains 75 adults, use the pigeonhole principle to explain why a group containing at least 11 of these people could be chosen so that all were born on the same day of the week. (3 marks)
- (b) In a certain community 40% of the people are males. If 15% of the males are grey-haired and 20% of the females are grey-haired, what percentage of the grey-haired people are males? (3 marks)

- (c) Determine how many numbers between 1 and 150 inclusive are multiples of 3, 4 or 5.
(4 marks)

Question 17

(8 marks)

A boat with a constant speed of 6 ms^{-1} is required to leave its mooring and motor directly to a jetty located 1 045 metres away on a bearing of 155° . A current of 1.5 ms^{-1} is running on a bearing of 200° .

(a) Sketch a diagram to that can be used to determine the direction the boat should steer. (2 marks)

(b) Determine the bearing that the boat should steer. (3 marks)

(c) Determine the time taken for the boat to reach the jetty. (3 marks)

Question 18

(7 marks)

(a) Let $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j}$, $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j}$ and $\mathbf{c} = c_1\mathbf{i} + c_2\mathbf{j}$.

Prove that the scalar product is distributive over vector addition: $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$.

(4 marks)

(b) Given $\mathbf{p} = 1.5\mathbf{i} + 4.5\mathbf{j}$, $\mathbf{q} = 3.5\mathbf{i} - 1.5\mathbf{j}$, $\mathbf{r} = -3.5\mathbf{i} + 5.5\mathbf{j}$ and $\mathbf{s} = -1.5\mathbf{i} + 1.5\mathbf{j}$,
determine $\mathbf{p} \cdot \mathbf{r} - \mathbf{p} \cdot \mathbf{s} + \mathbf{q} \cdot \mathbf{r} - \mathbf{q} \cdot \mathbf{s}$.

(3 marks)

Question 19

(8 marks)

A bag contains ten discs, three red, two green and one each of blue, yellow, black, white and brown.

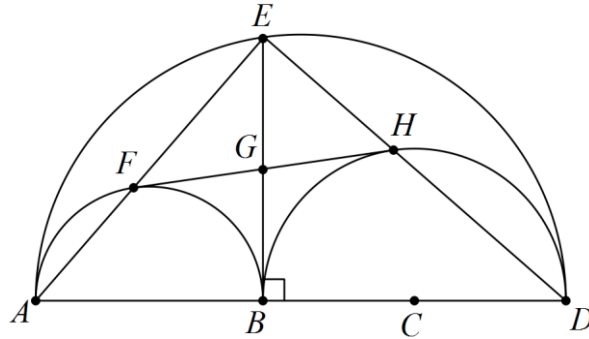
- (a) Determine the total number of selections there are of four of these discs. (Note: there are 5 different ways of selecting 4 discs from the bag) (4 marks)

- (c) Determine the total number of arrangements there are of four of these discs. (4 marks)

Question 20

(7 marks)

The diagram shows three semicircles with diameters AD , AB and BD , where B is a point on the diameter AD . Point C is the centre of the semicircle with diameter BD . Line BE is perpendicular to diameter AD and meets the largest semicircle at E . Points F and H are the intersections of lines AE and DE with the smaller semicircles. Point G is the intersection of lines FH and BE .



(a) Explain why $BFEH$ is a rectangle. (2 marks)

(b) Prove that $\triangle CBG$ and $\triangle CHG$ are congruent. (3 marks)

(c) Deduce that line FH is a tangent to the semicircle with diameter BD . (2 marks)

Additional working space

Question number: _____

Additional working space

Question number: _____

